

Analysis of Self-Gravitational Instability of Viscous, Magnetized, Two Component Plasma under the Effect of Radiative Heat-Loss Function

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ABSTRACT

The problem of Jeans instability of an infinite homogeneous self-gravitating, viscous, partially-ionized gaseous plasma carrying a uniform magnetic field in presence of thermal conductivity and the arbitrary radiative heat-loss function has been investigated. With the help of relevant linearized perturbation equations of the problem using normal mode analysis, a general dispersion relation is obtained and the dispersion relation is discussed for longitudinal propagation and transverse propagation separately. The condition of instability as well as stability of the fluid has been discussed. We found that the effect of viscosity, collision with neutrals and magnetic field have a stabilizing influence, while thermal conductivity has destabilizing influence on the Jeans instability.

Key words: - ISM (Interstellar medium), Viscosity, Thermal Conductivity, Arbitrary Radiative Heat-Loss Function, Partially Ionized Plasma and Collision Frequency.

1. INTRODUCTION

It is an established fact that the problem of self-gravitational instability is a broad area of research in the plasma physics, Astrophysics and into many other crucial phenomena of the interstellar medium (ISM). It plays an important role in star formation in magnetic dusty clouds via the gravitation collapsing process. Jeans (1929) have discussed the condition under which a

fluid becomes gravitational unstable under the action of its own gravity. Chandrashekhar (1961) has Presented a comprehensive survey of the various carried out on the gravitational instability problem under various assumptions. In this direction, many investigators have investigated the gravitational instability of a homogeneous plasma considering the effect of various parameter [Chhajlani and Sanghavi (1985),

Lenger (1978), Mamun (1998) and Kakati and Goswami (2000)].

Recently, Lima *et al.* (2002) have investigated the problem of Jeans gravitational instability and non extensive kinetic theory. Pensia *et al.* (2009) have discussed the problem of magneto-thermal instability of self-gravitating, viscous, Hall plasma in the presence of suspended particles. Pensia *et al.* (2010) have also investigated the role of magnetic field in contraction and fragmentation of interstellar clouds. Khan and Sheikh (2010) have discussed the instability of thermally conducting self-gravitating system. Thus we find that the problem of Jeans instability is the important phenomena to understand gravitational collapse of the protostar.

2. LINEARIZED PERTURBATION EQUATIONS

We assume that the two components of the partially ionized plasma (the ionized fluid and the neutral gas) behave like a continuum fluid and their state velocities are equal. The effect of the magnetic field, field of gravity and the pressure on the neutral components are neglected. Also it is assumed that the frictional force of the neutral gas on the ionized fluid is of the same order as the pressure gradient of the ionized fluid. Thus, we are considering only the mutual frictional effects between the neutral gas and the ionized fluid. It is assumed that the above medium is permeated with a uniform magnetic field $\vec{H} (0, 0, H)$.

Thus the linearized perturbation equations governing the motion of hydromagnetic thermally conducting two components of the partially ionized plasma are given by

$$\rho \frac{\partial \vec{v}}{\partial t} = -\vec{\nabla} \delta p + \vec{\nabla} \delta \varphi + \frac{1}{4\pi} (\nabla \times \vec{h}) \times \vec{H} + \rho_d v_c (\vec{v}_d - \vec{v}) + \nu (\nabla^2 \vec{v}) \quad (1)$$

$$\frac{\partial \vec{v}_d}{\partial t} = -v_c (\vec{v}_d - \vec{v}) \quad (2)$$

$$\frac{\partial \delta \rho}{\partial t} = -\rho \vec{\nabla} \cdot \vec{v} \quad (3)$$

$$\nabla^2 \delta \varphi = -4\pi G \delta \rho \quad (4)$$

$$\frac{1}{(\gamma-1)} \frac{\partial \delta p}{\partial t} - \frac{\gamma}{(\gamma-1)\rho} \frac{\partial \delta \rho}{\partial t} + \rho (\mathcal{L}_\rho \delta \rho + \mathcal{L}_T \delta T) - \lambda \nabla^2 \delta T = 0 \quad (5)$$

$$\frac{\delta p}{\rho} = \frac{\delta T}{T} + \frac{\delta \rho}{\rho} \quad (6)$$

$$\frac{\partial \vec{h}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{H}) \quad (7)$$

$$\vec{\nabla} \cdot \vec{h} = 0 \quad (8)$$

The parameters ϕ , G , λ , R , c , ρ , ρ_d , v_c , γ , T , ν and p denote the gravitational potential, gravitational constant, thermal conductivity, gas constant, velocity of light, density of ionized component, density of neutral components ($\rho \gg \rho_d$), collision frequency between two components, ratio of two specific heats, temperature, kinetic viscosity and pressure, respectively.

The perturbations in fluid velocity, fluid pressure, fluid density, magnetic field, gravitational potential, temperature and the radiative heat-loss function are given as $\vec{v}(v_x, v_y, v_z)$, δp , $\delta \rho$, $h(h_x, h_y, h_z)$, $\delta \phi$, δT , and $\delta \mathcal{L}$ respectively. In equation (5), $\mathcal{L}_{\rho, T}$

are the partial derivatives of the density dependent $\left(\frac{\partial \mathcal{L}}{\partial \rho}\right)_T$ and temperature dependent $\left(\frac{\partial \mathcal{L}}{\partial T}\right)_\rho$ heat-loss functions respectively.

DISPERSION RELATION

We assume that all the perturbed quantity vary as $\text{Exp. } \{i(k_x x + k_z z + \omega t)\}$ Where ω is the frequency of harmonic disturbances, $k_{x,z}$ are wave numbers in x and z direction, respectively, such that $k_x^2 + k_z^2 = k^2$ combining equation (5) and (6), we obtain the expression for δp as

$$\delta p = \left(\frac{\alpha + \sigma c^2}{\sigma + \beta}\right) \delta \rho \quad \Omega_v = \nu k^2, \quad \alpha = (\gamma - 1) \left(\mathcal{L}_T T - \mathcal{L}_\rho + \frac{\lambda k^2 T}{\rho}\right),$$

$$\beta = (\gamma - 1) \left(\frac{\mathcal{L}_T T \rho}{p} + \frac{\lambda k^2 T}{p}\right) \quad (9)$$

Where $\sigma = i\omega$, $c = \left(\frac{\gamma p}{\rho}\right)^{1/2}$ is the adiabatic velocity of sound in the medium

Using equation (2) – (9) in equation (1), we obtain the following algebraic equations for the amplitude components

$$\left(\sigma^2 + k_z^2 V^2 + \sigma \Omega_v + \frac{B v_c}{\sigma + v_c} \sigma^2\right) v_x + \frac{i k_x}{k^2} \sigma \Omega_T^2 s = 0 \quad (10)$$

$$\left(\sigma^2 + k^2 V^2 + \sigma \Omega_v + \frac{B v_c}{\sigma + v_c} \sigma^2\right) v_y = 0 \quad (11)$$

$$\left(\sigma + \Omega_v + \frac{B v_c}{\sigma + v_c} \sigma\right) v_z + \frac{i k_z}{k^2} \Omega_T^2 s = 0 \quad (12)$$

$$(i k_x k^2 V^2) v_x - \left(\sigma^3 + \frac{B v_c}{\sigma + v_c} \sigma^3 + \sigma^2 \Omega_v + \sigma \Omega_T^2\right) s = 0 \quad (13)$$

Where $s = \frac{\delta \rho}{\rho}$ is the condensation of the medium, $V = \frac{H}{(4\pi\rho)^{1/2}}$ is the Alfven velocity, $c^2 = \gamma c'^2$ where c and c' are the adiabatic and isothermal velocities of sound. Also we have assumed the following substitutions

$$B = \frac{\rho_d}{\rho}, \quad \Omega_T^2 = \frac{\sigma \Omega_j^2 + \Omega_I^2}{\sigma + \beta}, \quad \Omega_I^2 = k^2 \alpha - 4\pi G \rho, \quad \Omega_j^2 = k^2 c^2 - 4\pi G \rho,$$

$$R_1 = (1 + B) v_c + \Omega_v$$

$$M = a\sigma^2 + \sigma\Omega_T^2, \quad E' = a\sigma^2 + k_x^2 V^2, \quad E = \sigma a + K^2 V^2, \quad a = \sigma + \frac{Bv_c}{\sigma + v_c} \sigma + \Omega_v,$$

$$\Omega_k = \frac{\lambda_V k^2}{\rho c_p}$$

The nontrivial solution of the determinant of the matrix obtained from equation (10) to (13) with $\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z, \mathbf{S}$ having various coefficients that should vanish is to give the following dispersion relation $E E' a M + E' a k_x^2 V^2 \sigma \Omega_T^2 = 0$ (14)

The dispersion relation (14) shows the combined influence of thermal conductivity and arbitrary radiative heat-loss functions on the self gravitational instability of a two components of the partially-ionized plasmas we find that in this dispersion relation the terms due to the arbitrary radiative heat loss function with thermal conductivity have entered through the factor Ω_T^2 .

$$\left(\sigma + \frac{Bv_c}{\sigma + v_c} \sigma\right) \left(\sigma^3 + \frac{Bv_c}{\sigma + v_c} \sigma^3 + \sigma\Omega_T^2\right) \left(\sigma^2 + \frac{Bv_c}{\sigma + v_c} \sigma^2 + k^2 V^2\right) = 0 \quad (15)$$

We find that in the longitudinal mode of propagation the dispersion relation is modified due to the pressure of neutral particles, thermal conductivity and arbitrary radiative heat-loss functions. This dispersion relation has three independent factors, each represents the mode of propagation incorporating different parameters. The first factor of this dispersion relation equating to zero

$$\sigma^4 + \sigma^3(R_1 + \beta) + \sigma^2[v_c\{\Omega_v + (1+B)\beta\} + \Omega_j^2 + \beta\Omega_v] + \sigma[v_c\{\Omega_j^2 + \beta\Omega_v\} + \Omega_i^2] + v_c\Omega_i^2 = 0 \quad (17)$$

This dispersion relation for self gravitating fluid incorporated effect of neutral particles, viscosity, thermal conductivity and arbitrary radiative heat-loss function. It is evident from equation (17) that the condition of instability is independent of magnetic field. The

4. ANALYSIS OF THE DISPERSION RELATION

4.1. Longitudinal mode of propagation

For this case we assume all the perturbations longitudinal to the direction of the magnetic field i.e. ($k_z = k, k_x = 0$). This is the dispersion relation reduces in the simple form to give

$$\sigma^2 + \sigma R_1 + \Omega_v v_c = 0 \quad (16)$$

This represents the stable mode due to collision frequency

The second factor of equation (15) equating to zero, we obtain following dispersion relation

dispersion relation (17) is a fourth degree equation which may be reduced to particular cases so that the effect of each parameter is analyzed separately.

For thermally non-conducting, non-radiating, fully ionized fluid we have $\alpha = \beta = v_c = 0$ the dispersion relation (17) reduces to

$$\sigma^2 + \Omega_j^2 = 0 \text{ and } k < k_j = \left(\frac{4\pi G \rho}{c^2} \right)^{1/2} \quad (18)$$

The fluid is unstable for all Jeans wave number $k < k_j$. It is evident from equation (18) that Jeans criterion of

instability remains unchanged in the presence of neutral particles.

For non-arbitrary radiative heat loss function but thermally conducting and self gravitating fluid having neutral particles, the dispersion relation (17) reduces to

$$\sigma^4 + \sigma^3(R_1 + \Omega_k) + \sigma^2[\nu_c\{\Omega_v + (1+B)\Omega_k\} + \Omega_k\Omega_v + \Omega_{j1}^2] + \sigma[\nu_c(\Omega_{j1}^2 + \Omega_v\Omega_k) + \Omega_k\Omega_{j1}^2] + \nu_c\Omega_k\Omega_{j1}^2 = 0 \quad (19)$$

From equation (19) we get

$$k < k_{j1} = \left(\frac{4\pi G \rho}{c^2} \right)^{1/2} \quad (20)$$

Where k_{j1} is the modified Jeans wave number for thermally conducting system. It is clear from equation (20) that the Jeans length is reduced due to thermal conduction [as $\gamma > 1$], thus the system is destabilized. If we consider self-gravitating and thermally non-conducting plasma incorporated with neutral particles, arbitrary radiative heat-loss function then the condition of instability is given as

$$k < k_{j2} = k_j \left(\frac{\gamma \mathcal{L}_T}{\mathcal{L}_T - \rho \frac{\mathcal{L}_p}{T}} \right)^{1/2} \quad (21)$$

Where k_{j2} is the modified critical wave number due to inclusion of arbitrary radiative heat-loss function. Comparing equation (18) and (21) we find that the

critical wave number k_{j2} is very much different from the original Jeans wave number k_j and k_{j2} depends on derivatives of the arbitrary radiative heat-loss function with respect to local temperature $\mathcal{L}_T$ and local density $\mathcal{L}_p$ in the configuration. It is clear from equation (21) that when the arbitrary radiative heat-loss function is independent of density of the configuration (i.e. $\mathcal{L}_p = 0$), then $k_{j2} = k_j$ i.e. critical wave number remains unaffected and if the arbitrary radiative heat-loss function is independent of temperature ($\mathcal{L}_p = 0$), then k_{j2} vanishes.

The condition of instability of the system, when combined effect of all the parameters represented by the original dispersion relation (17) is given as

$$k_{j3} = \frac{1}{2^{1/2}} \left[\left\{ \frac{4\pi G \rho}{c^2} + \frac{\rho^2 \mathcal{L}_p}{\lambda T} - \frac{\rho \mathcal{L}_T}{\lambda} \right\} \pm \left\{ \left(\frac{4\pi G \rho}{c^2} + \frac{\rho^2 \mathcal{L}_p}{\lambda T} - \frac{\rho \mathcal{L}_T}{\lambda} \right)^2 + \frac{16\pi G \rho^2 \mathcal{L}_T}{\lambda c^2} \right\}^{1/2} \right]^{1/2} \quad (22)$$

Furthermore, if it is considered that the arbitrary radiative heat-loss function is purely density dependent ($\mathcal{L}_T = 0$) then the condition of instability is given as

$$k < k_{j4} = \left(\frac{4\pi G \rho}{c^2} + \frac{\rho^2 \mathcal{L}_p}{\lambda T} \right)^{1/2} \quad (23)$$

It is evident from inequality (22) that the critical wave number is increased or decreased, depending on whether the arbitrary radiative heat-loss function is an increasing or decreasing function of the density.

Now equating zero the third factor of equation (15) and after solving we obtain dispersion relation as

$$\sigma^6 + \sigma^5 A_1 + \sigma^4 A_2 + \sigma^3 A_3 + \sigma^2 A_4 + \sigma A_5 + A_6 = 0 \quad (24)$$

where

$$A_6 = \nu_e^4 V^4 k^4$$

$$R_2 = \nu_e (1 + B)$$

This dispersion relation shows the combined influence of magnetic field, viscosity and the effect of the neutral particles, but this mode is independent of thermal conductivity, arbitrary radiative heat-loss function and self gravitation. This equation gives Alfven mode modified by the dispersion effect of the neutral particles.

In the absence of neutral particles [$\nu_e = 0$], the dispersion relation (24) reduces to

$$\sigma^4 + 2\Omega_v \sigma^3 + \sigma^2 [\Omega_v + 2V^2 k^2] + \sigma [2\Omega_v V^2 k^2] + V^4 k^4 = 0 \quad (25a)$$

The necessary condition for stability of the system is that the equation (25-a) should have all coefficients positive, which is satisfied. The sufficient condition is that the Routh-Hurwitz criterion must be satisfied, according to which all the principal diagonal minors of the Hurwitz matrix must be positive for stable system. For the fourth degree equation (25-a) the four principal diagonal minors of Hurwitz matrix are positive as shown below.

$$\Delta_1 = 2\Omega_v > 0$$

$$\Delta_2 = [2\Omega_v^3 + 2\Omega_v V^2 k^2] > 0$$

$$\Delta_3 = 2\Omega_v^4 V^2 k^2 > 0$$

$$\Delta_4 = V^4 k^4 \Delta_3 > 0$$

We find that all Δ 's are positive, which shows that a magnetized viscous plasma is stable even in the absence of neutral particles. Thus, the equation (25-a) represents a stable Alfven mode modified by the dissipative effect of viscosity.

To examine the effect of magnetic field only, for simplicity we consider the system

for which $\Omega_v = 0$ and $\nu_e = 0$, then equation (25-a) reduces to

$$\sigma^4 + \sigma^2 2V^2 k^2 + V^4 k^4 = 0 \quad (25b)$$

$$\sigma_{1,2}^2 = -V^2 k^2$$

$$-\omega_{1,2}^2 = -V^2 k^2$$

$$\omega_{1,2} = \pm V k \quad (26)$$

Thus, we see in equation (26), the two Alfven waves moving in opposite direction.

4.2. Transverse propagation

For this case we assume all the perturbations are propagating perpendicular to the direction of the magnetic field, for, our convenience, we take $k_x = k$, and $k_z = 0$, the general dispersion relation (14) reduces to

$$a^2 [\sigma^2 a (\sigma a + V^2 k^2) + \sigma a \Omega_T^2] = 0 \quad (27)$$

This is the general dispersion relation for transverse propagation shows the combined influence of magnetic field, self gravitation, thermal conductivity presence of neutral particles and arbitrary radiative heat

loss function on the self-gravitational instability of a two components of the partially-ionized plasmas. Dispersion relation (27) has two distinct factors, each represents different mode of propagation

when equated to zero, separately. The first mode is same as discussed in the dispersion relation (16). The second factor of equation (27) equating zero and substituting the values of $\mathbf{a}$ and Ω_I^2 we get

$$\sigma^6 + \alpha_1 \sigma^5 + \alpha_2 \sigma^4 + \alpha_3 \sigma^3 + \alpha_4 \sigma^2 + \alpha_5 \sigma + \alpha_6 = 0 \quad (28)$$

Where $\alpha_1 = 2R_1 + \beta$

$$\alpha_2 = \Omega_I^2 + k^2 V^2 + \Omega_v^2 + 3v_e \Omega_v + 2R_1 \beta + v_e(1+B)(R_1 + \Omega_v) + R_7$$

$$\alpha_3 = \Omega_I^2 + k^2 V^2 (R_1 + \beta + v_e) + \Omega_I^2 v_e (3+B) + \beta v_e [3\Omega_v + (1+B)(R_1 + v_e) + 2\Omega_v R_1 v_e + R_7 (2v_e + \beta)]$$

$$\alpha_4 = R_7 (v_e^2 + 2\beta v_e) + \Omega_I^2 (R_1 + v_e) + v_e k^2 V^2 (R_1 + \Omega_v) + \Omega_I^2 v_e (R_1 + \Omega_v) + \Omega_v^2 v_e^2 + \beta [k^2 V^2 (R_1 + v_e) + 2\Omega_v R_1 v_e]$$

$$\alpha_5 = \beta v_e^2 R_7 + \Omega_I^2 \{v_e (R_1 + v_e) + \Omega_I^2 v_e^2 \Omega_v + v_e (\Omega_v v_e + \beta R_1) k^2 V^2 + \beta v_e \Omega_v (\Omega_v v_e + k^2 V^2)\}$$

$$\alpha_6 = \Omega_I^2 v_e^2 \Omega_v$$

This dispersion relation for transverse propagation shows the combined influence of thermal conductivity, viscosity, arbitrary radiative heat-loss function on the self-gravitation instability of two components partially-ionized plasmas with the effect of neutral particles. The condition of instability is obtained from dispersion relation (28) as

$$2 \left(1 + \frac{v^2}{c^2}\right) k_{j\mathbf{B}}^2 = \left[\left\{ \frac{4\pi G \rho}{c^2} + \frac{\rho^2 \mathcal{L}_\rho}{\lambda T} - \frac{\rho \mathcal{L}_T}{\lambda} \left(1 + \frac{v^2}{c^2}\right) \right\} \pm \left\{ \left(\frac{4\pi G \rho}{c^2} + \frac{\rho^2 \mathcal{L}_\rho}{\lambda T} - \frac{\rho \mathcal{L}_T}{\lambda} \left(1 + \frac{v^2}{c^2}\right) \right)^2 + \frac{16\pi G \rho^2 \mathcal{L}_T}{\lambda c^2} \left(1 + \frac{v^2}{c^2}\right) \right\}^{\frac{1}{2}} \right] \quad (29)$$

The medium is unstable for wave number $k < k_{j\mathbf{B}}$. It may be noted here that the critical wave number involves, derivative

of temperature dependent and density dependent arbitrary radiative heat-loss function, thermal conductivity of the medium and the magnetic field.

5. CONCLUSION

In this paper we have studied the wave propagation in an infinite homogeneous self-gravitating viscous and magnetized partially ionized fluid incorporating thermal conductivity and arbitrary heat-loss functions. The general dispersion relation is obtained, which is modified due to the presence of these parameters. This dispersion relation is reduced for longitudinal and transverse modes of propagation. We find that the Jeans condition remains valid but the expression of the critical Jeans wave number is modified. It is found that the system is stabilize in both the longitudinal and transverse modes of propagation due to the viscosity parameter. Owing to the inclusion of thermal conductivity the isothermal sound velocity is replaced by the adiabatic velocity of sound.

It is observed that, In the case of longitudinal propagation, the Alfvén mode is modified by the viscosity and collision frequency of neutrals with ionized gases. The effect of the collision with neutrals does not affect the instability of the fluid. The thermal conductivity has a destabilizing influence. It is also found that the density-dependent heat-loss function has a destabilizing influence on the instability of the fluid.

In the transverse mode of propagation, we find that the magnetic field has a stabilized influence on the self-gravitational instability of partially-ionized gaseous plasma. It is obtained that the gravitating thermal mode is affected by thermal conductivity and arbitrary radiative heat-loss functions. We also found the condition of instability and the expression of critical Jeans wave number both are

modified due to the presence of thermal conductivity, magnetic field and arbitrary radiative heat-loss function. It is observed that for an infinitely conducting fluid the condition of radiative instability is modified due to the presence of magnetic field and it shows a stabilizing influence. It is found that radiative critical wave number is same as original Alfvén critical wave number when the arbitrary radiative heat-loss functions are independent of density of the fluid. Thus the classical Jeans result regarding the rise of initial break up has been considerably modified due to the arbitrary radiative heat-loss function.

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